MACQUARIE UNIVERSITY

Department of Physics
Division of ICS

PHYS378 General Relativity and Cosmology (2005)

Assignment 2 due August 29

1. Confirm that the interval between two events is the same in all inertial reference frames.

2. For the following space displacement vectors, determine whether they are space-like, time-like or light-like:

   (a) $\Delta\vec{s} = 4\vec{e}_0 + 3\vec{e}_1$

   (b) $\Delta\vec{s} = -3\vec{e}_1 + 4\vec{e}_2$

   (c) $\Delta\vec{s} = 5\vec{e}_0 - 4\vec{e}_1 + 3\vec{e}_3$

   and determine what speed (as a fraction of the speed of light) a signal would have to travel in order to be present at two events separated by each of the above intervals.

3. Show, for the velocity four vector $\vec{u}$, that $\vec{u}\cdot\vec{u} = c^2$ and hence that $\vec{a}\cdot\vec{u} = 0$ where $\vec{a}$ is the acceleration four vector $\vec{a} = \dfrac{d\vec{v}}{d\tau}$.

4. It is quite simple to draw a world-line of a particle in the form of a closed loop (which would mean the particle travelling backward in time). Why is such a world-line not possible?

5. (a) What is wrong with the two expressions $a^\alpha = \Gamma^{\mu\nu}b_\nu$ and $a_\beta = \Gamma^{\mu\beta}a_\beta$.

   (b) Show that the two expressions $a^\mu b_\mu$ and $a^\nu b_\nu$ are identical by expanding the sum in each case.

   (c) Two four vectors $\vec{a}$ and $\vec{b}$ are related as follows:

   $$a^\mu = \Gamma^{\mu\nu}b_\nu \qquad b_\alpha = g_{\alpha\beta}b^\beta.$$

   Combine these two expressions to give the contravariant components of $\vec{a}$ in terms of the contravariant components of $\vec{b}$.

   (d) Given the only non-zero covariant components of the metric tensor g are $g^{00} = 1,\ g^{ii} = -1,\ i = 1, 2, 3$, calculate the covariant components $g_{\mu'\nu'}$ of the metric tensor g in another frame of reference S'.

   (e) The covariant components of the metric tensor g can be obtained from its contravariant components by lowering the indexies $g^{\mu\nu}$, i.e. $g_{\alpha\beta} = g_{\alpha\mu}g_{\beta\nu}g^{\mu\nu}$. By expanding out the implied sums, determine the contravariant components of g.

6. A certain second rank tensor T is defined by $\mathsf{T}(\vec{a}, \vec{b}) = (\vec{a}\cdot\vec{p})(\vec{b}\cdot\vec{v})$ where $\vec{p}$ is the momentum four vector and $\vec{v}$ the velocity four vector of a particle of rest mass m_0. Determine its covariant components $T_{\mu\nu}$, its mixed components $T_\mu{}^\nu$ and evaluate $T_\mu{}^\mu$.