MACQUARIE UNIVERSITY

Department of Physics
Division of ICS

PHYS378 General Relativity and Cosmology (2005)

Assignment 1 due August 18

1. Assume that one of the coordinate transformation equations for a reference frame S and a second reference frame S' moving at a velocity v_x with respect to each the first is of the form
$$x' = Ax^2 + Bt,$$
i.e. non-linear. Suppose that in the frame of reference S' a rod is lying at rest along the X' axis with one end at $x'_1 = 0$ the other at $x'_2 = l'$, i.e. the rod has a length l'. If the positions of the ends of the rod are measured at a time t in S, determine from this transformation law the coordinates x_1 and x_2 of the ends of the rod at this time t. Hence show that the length $x_2 - x_1$ of the rod as measured in S depends on when this length is measured.

2. A reference frame S' passes a second reference frame S with a velocity of $0.6c$ in the X direction. Clocks are adjusted in the two frames so that when $t = t' = 0$ the origins of the two reference frames coincide.

   (a) An event occurs in S with space-time coordinates $x_1 = 50$m, $t_1 = 2.0 \times 10^{-7}$s. What are the coordinates of this event in S'?

   (b) A second event occurs at $x_2 = 10$m, $t_2 = 3.0 \times 10^{-7}$ s. What are the coordinates of this event in S'?

   (c) What is the time interval between the events as measured in S and S'? Is this difference an example solely of time dilation? Give reasons for your conclusion.

3. Write down the Lorentz transformation equations that give the coordinate (x, t) in S of an event that has coordinates (x', t') in S' where S' is moving with a velocity v_x with respect to S. By treating these equations as a pair of simultaneous equations, invert the transformation by solving for x' and t' and interpret your result. Propose a quicker way of inverting the transformation.

4. A person at the origin of an inertial reference frame S observes a rod of proper length l_0 moving towards him at a speed v. He notes that the rod takes a time T to pass him.

   (a) As measured in S', the rest frame of the rod, how long will it take for the observer situated at the origin of S take to pass along the length of the rod? [Hint: answering this question does not require a knowledge of special relativity.]

   (b) Assuming that when the front end of the rod passes the observer, the clocks in S and in the rest frame S' of the rod are both set to read zero, fill in the missing information in the following space-time coordinates (x, t) and $x', t')$ of the two specified events:

      (i) Event E_1, the front end of the rod passing the observer at the origin of S, occurs at $(0, ?)$ in S and $(0, 0)$ in S'.

(ii) Event E_2, the tail end of the rod passing the observer at the origin of S, occurs at $(0, ?)$ in S and $(l_0, ?)$ in S'.

(c) Using the Lorentz transformation, establish a relation between the coordinates of event E_2.

(d) Hence prove that the velocity of the rod is given by

$$v = \frac{l_0/T}{\sqrt{1 + \left[(l_0/T)/c\right]^2}}$$

5. A fluorescent tube, stationary in a reference frame S, is arranged so as to light up simultaneously (in S) along its entire length l_0 at the time t. By considering as two simultaneous events in S the lighting up of two parts of the tube an infinitesimal distance Δx apart, determine the temporal and spatial separation of these two events in another frame of reference S' moving with a velocity v parallel to the orientation of the tube. Hence describe what is observed from this other frame of reference.

6. Two satellites are stationary in the S frame at points on the X-axis separated by a distance d. They fire laser pulses at one another simultaneously. From the point of view of the frame of reference of an observer space shuttle moving with a velocity u relative to S, show that one satellite fires a time $\gamma ud/c^2$ before the other.

7. An observer on earth sees two UFO's travelling directly towards each other with a velocity $0.7c$ relative to the observer on earth. According to an observer in one of the UFO's, how fast is the other UFO approaching? How fast is the distance between the UFO's diminishing according to the observer on earth?