PHYSICS 301 Quantum Mechanics (2009)

Quantum assignment 1

PHYS301 assignment 4

Due Date: 8 May 2009

1. (a) If light waves of wavelength λ pass through two slits separated by a distance d, then there forms an interference pattern whose intensity as a function of position on an observation screen a distance ℓ from the slits is given by

$$I(x) = I_1(x) + I_2(x) + 2\sqrt{I_1(x)I_2(x)}\cos\delta$$

where $I_n(x)$ is the intensity at position x on the observation screen due to waves emanating from slit n *only* (i.e. $I_n(x)$ is effectively the diffraction pattern due to the nth slit) and $\delta = 2\pi dx/\ell\lambda$.

(i) Suppose one slit is covered by a transparent material whose effect is to change the phase of the wave passing through that slit, i.e. so that $\delta \to \delta+\phi$ where ϕ is this new additional phase. Show that this has the effect of moving the position of the interference maxima.

(ii) Now suppose that the phase ϕ changes rapidly, and in a random way, between the extreme values 0 to 2π. As a consequence, $\cos\delta$ will also rapidly change its value. What is the average value of $\cos\delta$ as a result of these rapid changes in δ?

(iii) Derive an expression for the averaged intensity pattern on the observation screen in the presence of rapidly changing δ.

(b) Now consider the case of electrons of momentum p being fired at the two slits. What is the expression for the probability distribution function for the arrival of the electrons on the observation screen?

(c) Suppose the slits are continuously observed. This has the effect of introducing noise (decoherence) which randomly shifts the phase of the wave function of the electron at the positions of the slits. By analogy with the example above of light passing through the two slits, describe what the effect of this observation noise will be on the interference pattern on the observation screen. Comment on the result in comparison with what would be observed with bullets rather than electrons.

2. Bill, a not very conscientious physics student, is hired to watch the slits in a two slit experiment using electrons. Bill inserts a photographic plate used to record the electron strikes, peers through the microscope at the slits, gets bored, and falls asleep. He awakens just as his boss, Prof Catflap, returns. In a hurried display of initiative, Bill dashes off to develop the photograph. He is horrified to find that there is no interference pattern present, so he immediately and secretly replaces the plate with another he sees lying around which has such a pattern, and shows it to Prof Catflap. His boss fires him on the spot. What was Bill thinking when he made the replacement, and was Catflap justified in firing him?

3. The Feynman microscope apparatus used to determine which slit an electron passes through in a double-slit experiment is in general not perfect, as the image of the slits is smeared out in accordance with the resolution of the microscope. Thus, even if a photon is observed to strike the observation plate at a position consistent with the photon being scattered from an electron passing through, say, slit 1, there is a non-zero probability that the photon was in fact scattered from slit 2.

   Suppose that P is the probability that the photon was indeed scattered from the position of slit 1, so that immediately after the photon was detected, the electron is at the position of slit 1 (with $x = d/2$) with probability P, and at the position of slit 2 (with $x = -d/2$) with probability $1 - P$.

   (a) Calculate the mean position of the electron after the photon was detected.

   (b) Show that the standard deviation Δx in the position of the electron is $\Delta x = \sqrt{P(1-P)}d$, and hence determine the uncertainty Δp of the momentum of the electron in the x direction.

   (c) Because there is uncertainty Δp in the momentum in the x direction, the uncertainty Δx in the position of the electron will increase as the electrons move towards the final observation screen. Show that the uncertainty at the observation screen will be $\Delta x(t) = \Delta x + \Delta p t/m \geq \Delta x + \lambda\ell/(4\pi\Delta x)$ where t is the time of flight of the electrons to reach the observation screen, ℓ is the distance from the slits to this screen, and $\lambda = h/p$ is the de Broglie wavelength of the electron of momentum p. [Note: for Fraunhofer interference $\Delta x \ll \lambda\ell/(4\pi\Delta x)$.]

   (d) Consult your PHYS201 notes to confirm that the distance between an interference maximum and a neighbouring interference minimum of the electron interference pattern will be $\lambda\ell/2d$ for Fraunhofer interference. Hence find the probability P which would guarantee that the electron interference pattern is wiped out.

4. In the two slit experiment, it is found that at a point Q on an observation screen directly opposite the midpoint between the two slits, the probability of an electron striking Q if slit 2 is closed is P_1.

   (a) Assuming the source of electrons is symmetrically positioned with respect to the slits, what is the probability P_2 of an electron striking Q if slit 1 is closed?

   (b) What would be the probability of an electron striking Q if both slits were open, but the slit through which each electron passed was observed? Explain your reasoning.

   (c) What is the probability *amplitude* of an electron striking the point Q if both slits are open but the slit through which the electrons pass is *not* observed. Is the probability of an electron striking the point Q increased or decreased as compared to part (b). By what factor does this probability change?

   (d) At a second point Q' it is found that with both slits open, no electrons are seen to strike the observation screen and yet when the slit through which each electron passes is observed, electrons are observed to strike the screen at point Q'. By using an argument based on probability amplitudes, explain how can this occur.