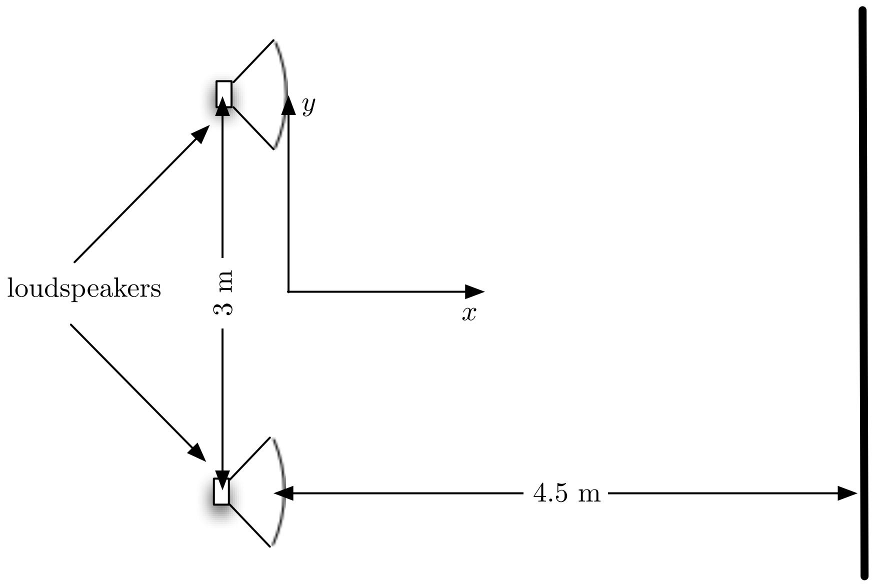
PHYSICS 201 Interference and Diffraction (2007)

Assignment 4

Due Date: 6 May 2009

1. Two loudspeakers are placed $d = 3$ m apart and facing the same direction. The loudspeaker diaphragms are vibrating in phase with each other, producing a single frequency sound with a wavelength of $\lambda = 1.2$ m.



The equation for the lines along which the interference pattern will have a maximum is given by

$$\frac{4y^2}{n^2\lambda^2} - \frac{4x^2}{d^2 - n^2\lambda^2} = 1$$

where $n = 0, 1, 2, \ldots$, but such that $n\lambda \leq d$, while the corresponding expression for the minima is found by replacing $n \to n + \frac{1}{2}$.

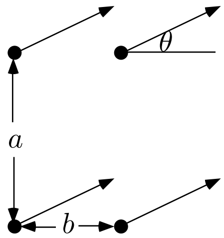
(a) Sketch in the lines along which there will be constructive interference of the sound from the two speakers, and where there will be destructive interference.

(b) Hence determine approximately the distance between the points on the wall on the right where the sound will be the loudest.

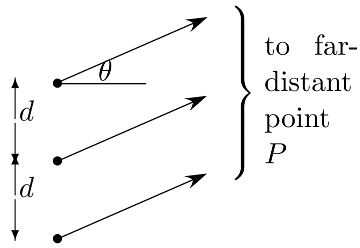
(c) Suppose there were only the speaker at $y = 1.5$m, and there was a perfectly reflecting wall along $y = 0$. Given that the sound reflected off the wall undergoes a phase change of π radians, where will the maxima and minima of the interference pattern be heard? Justify your answer.

(d) Why are no interference effects usually heard with speakers in a typical living room?

2. Four identical sources are located at the corners of a horizontal rectangle of sides a, b as shown. If they are all radiating in phase, at wavelength λ, what minimum values should a and b have to be able to produce maximum intensity in the direction $\theta = 30^\circ$ from the sources.



3. (a) Consider three sources S_1, S_2 and S_3 placed a distance d apart in which the middle source S_2 is oscillating a phase difference $\pi/2$ ahead of S_1, and S_3 a further phase difference of $\pi/2$ ahead of S_2. The source S_2 has twice the amplitude of the other two sources. Show that the total wave disturbance at a far distant point P can be written



$$y_P = a\mathrm{Im}\Big[e^{i(\omega t - kx)}\big\{1 + 2ie^{-i\delta} - e^{-2i\delta}\big\}\Big]$$

where 'Im[...]' means 'imaginary part', and where the phase difference δ can be expressed in terms of the wavelength λ of the field and geometrical quantities. Derive an expression for δ.

(b) Hence show that the resultant time averaged intensity $\overline{I}$ at P can be written

$$\overline{I} = \alpha \overline{I}_0 \cos^4(\tfrac{1}{2}\delta - \pi/4)$$

where $\overline{I}_0$ is the intensity due to source S_1 only. Determine the value of the factor α.

(c) By using calculus methods, show that there the interference pattern will have minima of zero for $\theta = 0, -\pi/6, -5\pi/6$, maxima for $\theta = \pi/6, 5\pi/6$ and non-zero minima for $\theta = \pm\pi/2$ for $d = \lambda/2$.

(d) Illustrate on a polar diagram the dependence of intensity on the direction θ.

[You may need

$$e^{ix} = \cos x + i \sin x, \quad \cos x = \frac{e^{ix} + e^{-ix}}{2}, \quad \text{and} \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i} \Big].$$

4. Light of wavelength 600 nm is normally incident on a screen in which there are 5 parallel slits equally spaced a distance 3 microns apart. The light forms an interference pattern on a screen a distance of 2.5m from the slits. Determine the position on the observation screen of the zeroth and first order principal maxima, the positions of the minima between these maxima, and the approximate position and intensity of any subsidiary maxima in between.