RESEARCH PROJECT:
Investigation of Relativistic Longitudinal Gauge Fields and their Interactions†

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Abstract

The ultimate aim of this project is to develop a mathematical theory which describes a new subclass of subatomic particles. The current classification of subatomic particles divides all existing particle types into two general classes, 1/2 integer spin particles called fermions, and (whole) integer spin particles called bosons. The new subclass of subatomic particles to be studied would add new boson particles to the physics inventory, or alternately reclassify some old ones. The key to obtaining this new subclass of bosons is the applicant's new "*relativistic longitudinal gauge*" which reduces a formally anti-Hermitian four-vector field to one effectively Hermitian component.

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I. OBJECTIVES AND OVERVIEW:

In the applicant's Ph.D. thesis [1], and in a recent article [2], new theorems were proved which allow one to distinguish between two types or subclasses of classical four-vector fields. In another article [3], the applicant developed further the classical relativistic Lagrangian and Hamiltonian formulation for these two classes of four-vector fields. The first of these two subclasses of four-vector fields is comprised of the usual electromagnetic-type four-vector fields in a "relativistic transverse gauge," i.e., the Lorentz gauge. The second subclass is comprised of new four-vector fields which are four-irrotational and satisfy what the applicant has called a "*relativistic longitudinal gauge*" [1–3].

These classical subclasses of four-vector fields lead in the quantum domain to two associated subclasses of quantum operator fields. In the case of the electromagnetic-type field, one eliminates an effectively anti-Hermitian scalar degree of freedom, the so-called scalar photon, by canceling it with the longitudinal photon via the Lorentz gauge constraint. The electromagnetic-type field is as a result a relativistic transverse field with two transverse photon states. Photons are spin one bosons. On the other hand, in the case of the new four-irrotational type four-vector field, in order to obtain physically reasonable results, one must start with a formally anti-Hermitian four-vector field. Then, application of the relativistic longitudinal gauge eliminates the three anti-Hermitian vector degrees of freedom, while retaining the effectively Hermitian scalar degree of freedom. This four-irrotational field is as a result a relativistic longitudinal gauge field with one effectively Hermitian scalar photon state, i.e., with a new associated potentially physical spin zero boson particle.

In this research project a canonical quantization procedure will be developed for this new subclass of four-vector fields. Then, a mathematical description of the interaction of this new subclass with its counterpart electromagnetic-type field subclass will be developed. The use of relativistic longitudinal gauge fields in quantum chromodynamics and in alternate (scalar-tensor) theories of gravitation will also be studied. This work will provide an important new paradigm for four-vector fields in particle field theory.

II. GAPS IN EXISTING KNOWLEDGE AND THE NEW METHODOLOGIES TO FILL THEM:

The first goal of this project is to complete a canonical quantization procedure for this new subclass of four-vector fields. This will be accomplished in the first year of the award. There exists a large body of work on the methodology of this procedure as applied to electromagnetic-type fields [4], but there is no existing canonical quantization procedure for this new subclass of four-vector fields. The present work will extend the existing procedure to this new case.

In the canonical quantization procedure for four-vector fields one first obtains harmonic oscillator modes in the classical domain and then one replaces them with quantum operators in what is called second quantization. It is at this point that the main changes in the quantization procedure will have to be made as appropriate for a relativistic longitudinal gauge field. Instead of applying the Lorentz gauge (where the four-divergence of the four-vector field is set to zero), which is the most appropriate relativistic constraint for Hermitian four-vector electromagnetic-type fields, one must choose a relativistic constraint which is appropriate for formally anti-Hermitian four-vector fields. This relativistic constraint is the new relativistic longitudinal gauge constraint where the Maxwell field tensor (the four-curl of the four-vector field), is set to zero. This makes the field four-irrotational, as opposed to four-solenoidal as in the electromagnetic-type field case, and is the defining difference between the two subclasses of four-vector fields [5]. A goal of this research project is to show that the relativistic longitudinal gauge constraint will yield one Hermitian-like scalar photon degree of freedom, while eliminating three anti-Hermitian vector photon degrees of freedom. New Hermitian-like degrees of freedom will imply that there exists a new (potentially physical) quantum particle subclass which could have observable consequences. Some of the details of the canonical quantization procedure as appropriate for an anti-Hermitian four-vector field are foreshadowed in the applicant's Ph.D. thesis [6] and in Ref. [3]. The version of the canonical quantization procedure that the applicant will follow, which emphasizes the differences between Hermitian and anti-Hermitian degrees of freedom in four-vector fields, historically derives from the Gupta-Bleuler quantization procedure for the electromagnetic field [7].

In the second year of the award I will focus on the description of interactions of relativistic longitudinal fields with electromagnetic-type fields, i.e., their boson-boson interactions. For example, the applicant would like to show that the enigmatic 1/2 quantum infinite zero-point vacuum contribution to the spin one electromagnetic field [8], which is currently ignored using normal ordering of quantum operators, can be canceled via a similar zero-point contribution to the relativistic longitudinal field which is of opposite sign. This would be important because the elimination of infinities in physical theories has been a principal goal of modern physics.

Another important aspect of the boson-boson interactions between relativistic longitudinal and transverse fields lies in the formalism itself. Although it has not been mentioned above, a Hermitian quantum operator corresponds to a pure-real valued classical field, while an anti-Hermitian quantum operator corresponds to a pure-imaginary valued classical field. It is believed that only pure-real valued field degrees of freedom lead to directly observable physical processes. The fact that the pure-imaginary valued classical four-vector field can lead to an effectively pure-real valued degree of freedom is therefore surprising. It is only the new relativistic longitudinal gauge introduced by the applicant which makes this possible [9]. In the context of the mixed boson-boson interactions under discussion, a point of view emerges where combined real and imaginary valued fields can be viewed as "true complex fields." This is not to be confused with the current usage of the words "complex fields" where one is for example actually referring to $U(1)$ gauge rotated pairs of pure-real valued fields. The theory of mixed boson-boson interactions when fully developed will therefore fill this gap in the formalism. From a pedagogical point of view this will contribute to a completion of the formalism of quantum field theory in an analogous way to the way in which the complex numbers complete the real numbers.

Other interactions with relativistic longitudinal bosons will also be explored. This is important because bosons, (if they are elementary, i.e., not made up of constituent parts), are considered to be the original transmitters of forces. The electromagnetic force is transmitted by the electromagnetic interaction boson, the photon, (actually two transverse photon states). If there are elementary bosons in this new subclass of four-vector fields they would likewise yield an interaction force with observable consequences. A likely candidate appears to follow from the pseudo four-vector version of the theory, which has nonzero interaction with a non-conserved pseudo four-current [3].

In the third year of the award, related topics will be studied. Foremost will be an attempt to develop a non-Abelian gauge version of the relativistic longitudinal gauge field. The research will focus on the extension of the four-irrotational constraint to non-Abelian gauges. Applications of the non-Abelian gauge version of the theory could include, for example, the question of whether or not one can replace quantum chromodynamical pure gauge fields, which coincidentally satisfy an extended relativistic longitudinal gauge, with this new subclass of fields. Currently, these pure gauge boson fields, referred to as "ghost" particles, are enigmatically labelled as fermions due to the sign of the contributions to their Lagrangian [10]. A reclassification of these states as relativistic longitudinal fields would restore the boson nature of pure gauge fields to these particles, eliminating a current paradox.

Another topic would be to explore the use of a relativistic longitudinal gauge four-vector field, with its single potentially physical scalar boson and its ostensibly attractive-type force law between like charged pure imaginary valued particles, as part of a (scalar-tensor) gravitational field model. While Einstein's general theory of relativity provides a very successful classical field theory [11], at present its quantum unification with the other forces of nature is fraught with difficulties [12] and no fully successful "unified field theory" has been formulated. This new class of relativistic longitudinal gauge four-vector fields, as well as the possibility of a new class of irrotational Hermitian degrees of freedom following analogously from spin 2 gravitational field theories, may provide new insight into the formulation of a quantum theory of gravity.

In conclusion, since this new subclass of subatomic particles is not at present classified, the development of a complete understanding of its mathematical properties will fill a major gap in existing knowledge. This is the ultimate objective of this theoretical and mathematical physics research project.

III. PRELIMINARY WORK:

Much of the preliminary work for this project is contained in the applicant's Ph.D. thesis [1]. A related article appears in the Journal of Mathematical Physics (JMP) [2]. Another related article appearing in JMP has obtained new results on pure imaginary valued pseudo four-vector fields [3].

In addition to the results already mentioned in Sec. II, it should also be mentioned that the applicant's Ph.D. thesis and related journal article [2] demonstrated a correspondence with a well known theorem in physics. It was shown that a three-vector Helmholtz identity could be obtained from one of the applicant's four-vector theorems as a non-relativistic Newtonian limit. Associated uniqueness theorems for four-vector fields were also obtained. In the process, new scalar field identities and uniqueness theorems followed as well. That this Helmholtz identity follows as a special case of one of the applicant's theorems is further evidence that a new more comprehensive understanding of four-vector fields has been obtained.

The applicant's Ph.D. thesis was primarily confined to classical field theory. However, as mentioned in Sec. II, the application of the resulting subclass of four-vector fields to the quantum domain was also foreshadowed. This was continued in Ref. [3]. The quantum domain study is the natural next step in the development of a complete particle field theory and this will be taken up in earnest in the course of this project.

An article on the canonical quantization procedure for this new subclass of four-vector fields in the relativistic longitudinal gauge is already under preparation. An example of a single (non-physical) anti-Hermitian scalar field has been included in this article so that the tacit assumptions of quantum field theory for the anti-Hermitian field case can be fully explored. In order to be consistent it turns out that a field Lagrangian density must be used which is simply quadratic in the fields in order for an anti-Hermitian scalar field operator to yield an indefinite metric Fock space. The counter-example of a field Lagrangian density which is quadratic in the modulus of the fields yields a definite metric Fock space for an anti-Hermitian scalar field, which is a contradiction. This seemingly innocuous result paves the way for the canonical quantization of anti-Hermitian four-vector fields in the relativistic longitudinal gauge.

IV. TIMETABLE:

The following is a timetable summarizing the new procedures and methodologies to be developed as already described in more detail in Sec. II:

•*First Year:* In the first year of the award, a canonical quantization procedure for this new subclass of four-vector fields in the relativistic longitudinal gauge will be completed. A modified Gupta-Bleuler quantization procedure will be considered, as appropriate for fields with mixed Hermitian and anti-Hermitian degrees of freedom. The successful development of this new procedure will require the integration of the relativistic longitudinal gauge constraint equations with certain aspects of gauge transformation invariance. In electromagnetic-type theories, gauge invariance combines naturally with the Lorentz gauge constraint to reduce the number of degrees of freedom to two Hermitian degrees of freedom. A similar synthesis will need to be implemented in order to obtain the expected single Hermitian degree of freedom for this new subclass of four-vector fields as already described in Sec. II.

•*Second Year:* In the second year of the award, a mathematical description of the boson-boson interactions between relativistic longitudinal and transverse fields will be developed. A standard method will be used which combines the Lagrangians of one representative example from each of the two subclasses of four-vector fields. It is a goal of this project to show that this combination of four-vector field subclasses leads to fortuitous cancellations of infinities in the formalism. This may result in interesting renormalization counter terms. In the process a pedagogical overview of the new formalism will be obtained. Interactions between relativistic longitudinal bosons and fermions will also be studied in order to obtain an interaction force description. An interaction force would imply that there may be elementary subatomic particles associated with this new subclass of four-vector fields. Also, the existence of a pure imaginary valued pseudo-charge four-current [3] would imply that there may be fermions with "complex charge". A theory along the lines of Cabibbo and Ferrari [13] or Schwinger [14] could then be constructed using two four-vector potentials. A Dirac-like charge quantization condition for the complex charge would then be implied. There also appears to be a connection with the chiral invariance of the massless Dirac equation. [15] Consequently, upon first inspection, the gauge group $U(1)_V \otimes U(1)_A$ appears to be the appropriate one for the description of the interaction of pure-real valued vector and pure-imaginary valued axial vector currents.

• *Third Year:* In the third year of the award, related topics will be studied. First, a non-Abelian gauge version of the relativistic longitudinal gauge will be studied. The standard method for extending four-vector fields to higher dimensional non-Abelian gauges will be used. The primary gauge constraint equation, in this case the relativistic longitudinal gauge constraint, will be replaced by its expected higher dimensional non-Abelian counterpart. $SU(2)$ and $SU(3)$ versions of the theory will be introduced. This will be a preliminary investigation with the goal of evaluating whether or not so-called "ghost" particles can be described better by relativistic longitudinal pure gauge fields.

A secondary topic involving the application of relativistic longitudinal gauge fields to gravitational field theory may also be studied. It may be possible to obtain new irrotational Hermitian degrees of freedom from a spin 2 gravitational field theory in an analogous way as in the four-vector field case. Interesting renormalization counter terms which cancel the infinite zero point energies of the fields may be suggested. The ostensibly attractive type force law of a relativistic longitudinal gauge (scalar) boson may even fit into a scalar-tensor type theory of gravitation, resulting in a new pseudo-vector-tensor theory.

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[5] See Ref. [1], Secs. 1.1, 1.2, 3.3, & 3.4; or Ref. [2], Secs. II A & II B, and Ref. [3], Secs. III C & III D.

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[8] See for example pp. 45-46, 87 in F. Mandl and G. Shaw, loc. cit.

[9] See Ref. [1], Sec. 3.1; or Ref. [3], Sec. III A.

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