Chapter 4

The Two Slit Experiment

This experiment is said to illustrate the essential mystery of quantum mechanics[1]. This mystery is embodied in the apparent ability of a system to possess properties which, from a classical physics point-of-view, are mutually contradictory. We have already touched on one such instance, in which the same physical system can exhibit under different circumstances, either particle or wave-like properties, otherwise known as wave-particle duality. It is this that is examined in greater detail here.

The experiment will be considered in three forms: performed with macroscopic particles, with waves, and with electrons. The first two experiments merely show what we expect to see based on our everyday experience. It is the third which displays the counterintuitive behaviour of microscopic systems – a peculiar combination of particle and wave like behaviour which cannot be understood in terms of the concepts of classical physics. The analysis of the two slit experiment presented below is more or less taken from Volume III of the Feynman Lectures in Physics.

4.1   An Experiment with Bullets

Imagine an experimental setup in which a machine gun is spraying bullets at a screen in which there are two narrow openings, or slits which may or may not be covered. Bullets that pass through the openings will then strike a further screen, the detection or observation screen, behind the first, and the point of impact of the bullets on this screen are noted.

Suppose, in the first instance, that this experiment is carried out with only one slit opened, slit 1 say. A first point to note is that the bullets arrive in 'lumps', (assuming indestructible bullets), i.e. every bullet that leaves the gun arrives as a whole somewhere on the detection screen. Not surprisingly, what would be observed is the tendency for the bullets to strike the screen in a region somewhere immediately opposite the position of the open slit, but because the machine gun is firing erratically, we would expect that not all the bullets would strike the screen in exactly the same spot, but to strike the screen at random, though always in a region roughly opposite the opened slit. We can represent this experimental outcome by a curve $P_1(x)$ which is simply such that

$$P_1(x)\delta x = \text{ probability of a bullet landing in the range } (x, x + \delta x). \qquad (4.1)$$

---

[1]Another property of quantum systems, known as 'entanglement', sometimes vies for this honour, but entanglement relies on this 'essential mystery' we are considering here

If we were to cover this slit and open the other, then what we would observe is the tendency for the bullets to strike the screen opposite this opened slit, producing a curve $P_2(x)$ similar to $P_1(x)$. These results are indicated in Fig. (4.1).


Figure 4.1: The result of firing bullets at the screen when only one slit is open. The curves $P_1(x)$ and $P_2(x)$ give the probability densities of a bullet passing through slit 1 or 2 respectively and striking the screen at x.

Finally, suppose that both slits are opened. We would then observe the bullets would sometimes come through slit 1 and sometimes through slit 2 – varying between the two possibilities in a random way – producing two piles behind each slit in a way that is simply the sum of the results that would be observed with one or the other slit opened, i.e.

$$P_{12}(x) = P_1(x) + P_2(x) \tag{4.2}$$

Figure 4.2: The result of firing bullets at the screen when both slits are open. The bullets accumulate on an observation screen, forming two small piles opposite each slit. The curve $P_{12}(x)$ represents the probability density of bullets landing at point x on the observation screen.
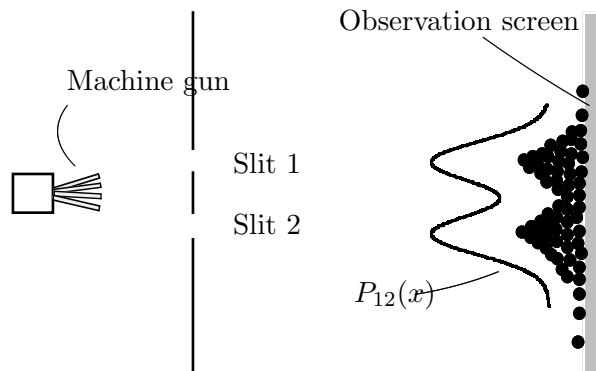
4.2   An Experiment with Waves

Now repeat the experiment with waves. For definiteness, let us suppose that the waves are light waves of wavelength λ. The waves pass through the slits and then impinge on the screen where we measure the intensity of the waves as a function of position along the screen.

First perform this experiment with one of the slits open, the other closed. The resultant intensity distribution is then a curve which peaks behind the position of the open slit, much like the curve obtained in the experiment using bullets. Call it $I_1(x)$, which we know is just the square of the amplitude of the wave incident at x which originated from slit 1. If we deal only with the electric field, and let the amplitude[2] of the wave at x at

---

[2]The word 'amplitude' is used here to represent the value of the wave at some point in time and space, and is *not* used to represent the maximum value of an oscillating wave.

time t be $E(x,t) = E(x)\exp(-i\omega t)$ in complex notation, then the intensity of the wave at x will be

$$I_1(x) = |E_1(x,t)|^2 = E_1(x)^2. \tag{4.3}$$

Close this slit and open the other. Again we get a curve which peaks behind the position of the open slit. Call it $I_2(x)$. These two outcomes are illustrated in Fig. (4.3)

Figure 4.3: The result of directing waves at a screen when only one slit is open. The curves $I_1(x)$ and $I_2(x)$ give the intensities of the waves passing through slit 1 or 2 respectively and reaching the screen at x. (They are just the central peak of a single slit diffraction pattern.)

Now open both slits. What results is a curve on the screen $I_{12}(x)$ which oscillates between maxima and minima – an interference pattern, as illustrated in Fig. (4.4). In fact, the theory of interference of waves tells us that

$$
\begin{aligned}
I_{12}(x) =& |E_1(x,t) + E_2(x,t)|^2 \\
=& I_1(x) + I_2(x) \\
&+ 2E_1 E_2 \cos\left(\frac{2\pi d \sin\theta}{\lambda}\right) \\
=& I_1(x) + I_2(x) \\
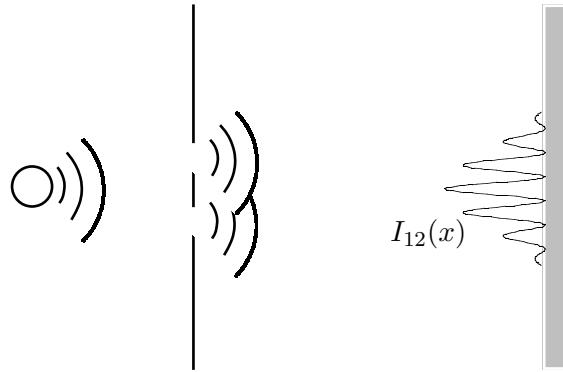&+ 2\sqrt{I_1(x)I_2(x)}\cos\delta
\end{aligned}
\tag{4.4}
$$

Figure 4.4: The usual two slit interference pattern.

where $\delta = 2\pi d \sin\theta/\lambda$ is the phase difference between the waves from the two slits arriving at point x on the screen at an angle θ to the straight through direction. This is certainly quite different from what was obtained with bullets where there was no interference term. Moreover, the detector does not register the arrival of individual lumps of wave energy: the intensity can have any value at all.

4.3   An Experiment with Electrons

We now repeat the experiment for a third time, but in this case we use electrons. Here we imagine that there is a beam of electrons incident normally on a screen with the two slits, with all the electrons having the same energy E and momentum p. The screen is a fluorescent screen, so that the arrival of each electron is registered as a flash of light – the signature of the arrival of a *particle* on the screen. It might be worthwhile pointing out that the experiment to be described here was not actually performed until the very recent past, and even then not quite in the way described here. Nevertheless, the

conclusions reached are what would be expected on the basis of what is now known about quantum mechanics from a multitude of other experiments. Thus, this largely hypothetical experiment (otherwise known as a thought experiment or gedanken experiment) serves to illustrate the kind of behaviour that quantum mechanics would produce, and in a way that can be used to establish the basic principles of the theory.

Let us suppose that the electron beam is made so weak that only one electron passes through the apparatus at a time. What we will observe on the screen will be individual point-flashes of light, and only one at a time as there is only one electron passing through the apparatus at a time. In other words, the electrons are arriving at the screen in the manner of particles, i.e. arriving in lumps. If we close first slit 2 and observe the result we see a localization of flashes in a region directly opposite slit 1. We can count up the number of flashes in a region of size δx to give the fraction of flashes that occur in the range x to $x + \delta x$, as in the case of the bullets. As there, we will call the result $P_1(x)$. Now do the same with slit 1 closed and slit 2 opened. The result is a distribution described by the curve $P_2(x)$. These two curves give, as in the case of the bullets, the probabilities of the electrons striking the screen when one or the other of the two slits are open. But, as in the case of the bullets, this randomness is not to be seen as all that unexpected – the electrons making their way from the source through the slits and then onto the screen would be expected to show evidence of some inconsistency in their behaviour which could be put down to, for instance, slight variations in the energy and direction of propagation of each electron as it leaves the source.

Now open both slits. What we notice now is that these flashes do not always occur at the same place – in fact they appear to occur randomly across the screen. But there is a pattern to this randomness. If the experiment is allowed to continue for a sufficiently long period of time, what is found is that there is an accumulation of flashes in some regions of the screen, and very few, or none, at other parts of the screen. Over a long enough observation time, the accumulation of detections, or flashes, forms an interference pattern, a characteristic of wave motion i.e. in contrast to what happens with bullets, we find that, for electrons, $P_{12}(x) \neq P_1(x) + P_2(x)$. In fact, we obtain a result of the form

$$P_{12}(x) = P_1(x) + P_2(x) + 2\sqrt{P_1(x)P_2(x)} \cos \delta \qquad (4.5)$$

so we are forced to conclude that this is the result of the interference of two waves propagating from each of the slits. One feature of the waves, namely their wavelength, can be immediately determined from the separation between successive maxima of the interference pattern. It is found that $\delta = 2\pi d \sin\theta/\lambda$ where $\lambda = h/p$, and where p is the momentum of the incident electrons. Thus, these waves can be identified with the de Broglie waves introduced earlier, represented by the wave function $\Psi(x, t)$.

So what is going on here? If electrons are particles, like bullets, then it seems clear that the electrons go *either* through slit 1 *or* through slit 2, because that is what particles would do. The behaviour of the electrons going through slit 1 should then not be affected by whether slit 2 is opened or closed as those electrons would go nowhere near slit 2. In other words, we have to expect that $P_{12}(x) = P_1(x) + P_2(x)$, but this not what is observed. It appears that we must abandon the idea that the particles go through one slit or the other. But if we want to retain the mental picture of electrons as particles, we must conclude that the electrons pass through *both* slits in some way, because it is only by 'going through both slits' that there is any chance of an interference pattern forming. After all, the interference term depends on d, the separation between the slits, so we must expect that the particles must 'know' how far apart the slits are in order for the positions that they strike the screen to depend on d, and they cannot 'know' this if each electron goes through only one slit.

We could imagine that the electrons determine the separation between slits by supposing that they split up in some way, but then they will have to subsequently recombine before striking the screen since all that is observed is single flashes of light. So what comes to mind is the idea of the electrons executing complicated paths that, perhaps, involve them looping back through each slit, which is scarcely believable. The question would have to be asked as to why the electrons execute such strange behaviour when there are a pair of slits present, but do not seem to when they are moving in free space. There is no way of understanding the double slit behaviour in terms of a particle picture only.

We may argue that one way of resolving the issue is to actually monitor the slits, and look to see when an electron passes through each slit. This could be done, for instance, by shining a light on each of the slits. If an electron goes through a slit, then it scatters some of this light, which can be observed with a microscope. We immediately know what slit the electron passed through, but unfortunately, as a consequence of gaining this knowledge, what is found is that the interference pattern disappears, and what is seen on the screen is the same result as for bullets.

We can construct an argument, illustrated in Fig. (4.5) due to Feynman, but based on an idea originally used by Heisenberg when he first discovered the uncertainty relation, that 'explains' this result. The essence of the argument is that the scattering of light by an electron also produces a recoil in the motion of the electron: it will be deflected from its path. However, the deflection cannot be made arbitrarily small. By a conspiracy of physical requirements, this deflection will always be sufficient to guarantee that an electron that might have originally been heading towards a maximum of the interference pattern will be deflected by an amount that can be enough to result in the electron heading towards the position of one of the minima, so that the pattern is washed out. But what is this 'conspiracy of physical requirements'? First is the requirement that the image in the microscope be sufficiently well-resolved that it is possible to unequivocally say from which slit the observed photon was scattered.

Figure 4.5: An electron with momentum p passing through slit 1 scatters a photon which is observed through a microscope. The electron is gains momentum Δp and is deflected from its original course.

To see what this means, suppose we forget for the present about firing electrons through the slits, and simply imagine that we are looking at the two slits through a microscope, with the slits illuminated by light of wavelength λ_l. Because of the finite aperture of the microscope lens, these two images (produced on a photographic plate, for instance, or on the retina of the experimenter's eye) will be somewhat fuzzy and will tend to overlap, but as the wavelength is made shorter, the images become sharper. In order to be able to distinguish the two images, the classical theory of optical imaging tells us that the wavelength of the light must be no more than

$$\lambda_l = 2d \sin \alpha \tag{4.6}$$

where 2α is the angle of the apex of a cone formed by the microscope lens at the position of the slits. This condition is the first element in this 'conspiracy'. Now suppose that the light is made to shine on the slits with the aim of trying to see, through the microscope,
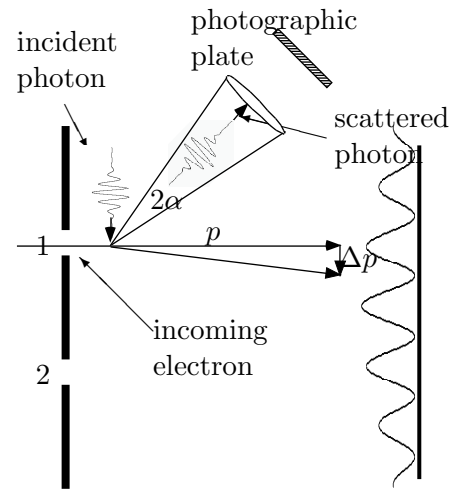
light that is scattered by an electron as it passes through one slit or the other. Since we want to minimize the effect that this light might have on the electrons, that is, to deflect the electrons as little as possible from their original path, we can suppose that the intensity of the light is turned down as much as possible. But it is in doing this that the second element in the conspiracy comes about: the light will be seen to arrive in small 'packages' – photons, each of which will carry a momentum $p = h/\lambda_l$. What we will then see through the microscope will be the arrival (registered as small dots on a photographic plate) of individual photons scattered by the electrons. These dots will be seen occurring in the field of view at positions where the image of one or the other slit was earlier determined to be. In this way, we can say where the electron was positioned when it scattered the photon – near one or the other of the two slits. But, in order for the photon to strike the photographic plate at all, it must be scattered somewhere into the cone subtended by the microscope lens at the position of the slits, i.e. a photon scattered from an electron passing through, say, the upper slit, could enter the microscope lens along a path that ranges anywhere over an angular range of 2α and still be seen to arrive at the position of the image of the upper slit. If we analyze this scattering process, and take into account that each photon carries with it a momentum of h/λ_l, we can readily show by use of conservation of momentum arguments, that, depending on the amount by which the photon is deflected over this angular range of 2α, there will be momentum transferred to any electron passing through a slit which can result in a change in momentum of the electron of as much as $\Delta p \sim 2h \sin \alpha/\lambda_l$ at right angles to the original direction of motion of the electron. Thus, an electron passing through, say, the upper slit, could be deflected by an angle of up to $\Delta\theta$ where

$$\Delta\theta \sim \Delta p/p \sim \lambda_l/d. \tag{4.7}$$

and the photon still being seen as coming from the position of this slit. But since the angular separation between a maximum of the interference pattern and a neighbouring minimum is $\lambda_l/2d$, this immediately tells us that the deflection is at least enough to displace an electron from its path heading for a maximum of the interference pattern, to one where it will strike the screen at the position of a minimum. The overall result is to wipe out the interference pattern. Thus, by monitoring an explicitly particle characteristic of the electron, i.e. where it is, the experiment yields the results that would be found with particles.

This argument is valuable in that it shows how nature 'conspires' to guarantee that the uncertainty principle is always obeyed. However, such an argument is flawed as it tries to explain the behaviour observed here by using classical physics to describe the collision, and quantum physics to provide us with the momenta of the participants. In any case, there is always something left unexplained. We could ask why it is that the photons could be scattered through a range of angles – after all, the experiment could be set up so that every photon sent towards the slits has the same momentum every time, and similarly for every electron, so every scattering event should produce the same result. But instead, what we find is that the photons, after passing through the microscope do not always produce a dot at the same position on the photographic plate – these arrivals occur at random positions and the situation starts to look a lot like what we had before, though instead of the difficulty of deciding through which slit the electron passes, what we discover is that we now have to deal with the difficulty of deciding through which part of the lens of the microscope that the scattered photons pass!! The 'mystery', instead of being explained, is simply displaced somewhere else.

As far as it is known, *any* experiment that can be devised – either a real experiment or a gedanken (i.e. a thought) experiment – that attempts to determine which slit the

electron passes through always results in the disappearance of the interference pattern. An 'explanation' of why this occurs could perhaps be constructed in each case, but in any such explanation, the details of the way the physics conspires to produce this result will differ from one experiment to the other [3]. But these explanations invariably require a mixture of classical and quantum concepts, i.e. they have a foot in both camps and as such are not entirely satisfactory. And in any case, the mystery is never ever explained, merely replaced with another. Moreover, in recent times, it has become apparent that the experiments do not even need to involve any direct interaction between the electrons and the apparatus used to monitor the slits, so there is no place for the kind of quantum/classical explanation given above. What is learned from this is that if we *gain knowledge* by whatever means, of which slit the electron passes through or, in other words, if we *know* which slit the electron goes through, then the observed pattern of the screen is the same as found with bullets: no interference, with $P_{12}(x) = P_1(x) + P_2(x)$. Confirming that the electrons definitely go through one slit or another, which is a property that we expect particles to possess, then results in the electrons behaving as particles do. If we do not look, then we are left with the uncomfortable thought that in some sense each electron passes through both slits, resulting in the formation of an interference pattern, the signature of wave motion. Thus, the electrons behave either like particles or like waves, depending on what it is that is being observed, a dichotomy that is known as wave-particle duality.

The fact that *any* experiment that determines which slit the electron passes through always results in the disappearance of the interference pattern suggests that there is a deep physical principle at play here – a law or laws of nature – that overrides, irrespective of the physical setup, any attempt to both watch where the particles are, and to observe the interference effects. The laws are the laws of quantum mechanics, and one of their consequences is the uncertainty principle, that is $\Delta x \Delta p \geq \frac{1}{2}\hbar$. It can be argued that pinning down the position of an electron well enough to say that it is passing through a particular slit amounts to specifying its x position to within $\Delta x \lesssim d/2$, where d is the separation of the slits. But doing so implies that there is an uncertainty in the sideways momentum of the electron given by $\Delta p \gtrsim \hbar/d$. The argument then proceeds as for the Heisenberg microscope, that is, since the electrons have a total momentum $p = h/\lambda$, this amounts to a change in direction through an angle

$$\Delta \theta = \frac{\Delta p}{p} \gtrsim \frac{\lambda}{2\pi d} \tag{4.8}$$

As the angular separation between a minimum and a neighbouring maximum of the diffraction pattern is $\lambda/2d$, it would appear that the estimate of the uncertainty in the sideways momentum Δp arising from trying to observe through which slit the particle passes is a factor of $1/\pi$ too small, and so is not enough to guarantee that the interference pattern would be washed out. The difficulty is that the estimate provided by the uncertainty principle here is a significant underestimate of Δp. The best that can probably be said is that the uncertainty principle indicates the kind of behaviour that is to be expected, but does not provide a good estimate, in this case, of the kind of effect that is observed.

Using the uncertainty principle in this way does not supply a 'physical' mechanism for why the interference pattern washes out, i.e. no physical mechanism appears to be at play, only the abstract requirements of the uncertainty principle. Nothing is said about how

---

[3]Einstein, who did not believe quantum mechanics was a complete theory, played the game of trying to find an experimental setup that would reveal a flaw in the position-momentum uncertainty relation. Bohr answered all of Einstein's challenges, including, in one instance, using Einstein's own theory of general relativity to defeat a proposal of Einstein's. It was at this point that Einstein abandoned the game, but not his attitude to quantum mechanics.

the position of the particle is pinned down to within an uncertainty Δx. It is the laws of quantum mechanics (from which the uncertainty principle follows) that tell us that the interference pattern must disappear if we measure particle properties of the electrons, and this is so *irrespective of the particular kind of physics involved in the measurement* – the individual physical effects that may be present in one experiment or another are subservient to the laws of quantum mechanics.

4.4   Probability Amplitudes

First, a summary of what has been seen so far. In the case of waves, we have seen that the total amplitude of the waves incident on the screen at the point x is given by

$$E(x,t) = E_1(x,t) + E_2(x,t) \tag{4.9}$$

where $E_1(x,t)$ and $E_2(x,t)$ are the waves arriving at the point x from slits 1 and 2 respectively. The intensity of the resultant interference pattern is then given by

$$
\begin{aligned}
I_{12}(x) =& |E(x,t)|^2 \\
=& |E_1(x,t) + E_2(x,t)|^2 \\
=& I_1(x) + I_2(x) + 2E_1 E_2 \cos\left(\frac{2\pi d \sin\theta}{\lambda}\right) \\
=& I_1(x) + I_2(x) + 2\sqrt{I_1(x)I_2(x)} \cos\delta
\end{aligned}
\tag{4.10}
$$

where $\delta = 2\pi d \sin\theta/\lambda$ is the phase difference between the waves arriving at the point x from slits 1 and 2 respectively, at an angle θ to the straight through direction.

The point was then made that the probability density for an electron to arrive at the observation screen at point x had the same form, i.e. it was given by the same mathematical expression

$$P_{12}(x) = P_1(x) + P_2(x) + 2\sqrt{P_1(x)P_2(x)} \cos\delta. \tag{4.11}$$

so we were forced to conclude that this is the result of the interference of two waves propagating from each of the slits. Moreover, the wavelength of these waves was found to be given by $\lambda = h/p$, where p is the momentum of the incident electrons so that these waves can be identified with the de Broglie waves introduced earlier, represented by the wave function $\Psi(x,t)$.

Thus, we are proposing that incident on the observation screen is the de Broglie wave associated with each electron whose total amplitude at point x is given by

$$\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t) \tag{4.12}$$

where $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are the amplitudes at x of the waves emanating from slits 1 and 2 respectively. Further, since $P_{12}(x)\delta x$ is the probability of an electron being detected in the region $x, x + \delta x$, we are proposing that

$$|\Psi(x,t)|^2 \delta x \propto \text{probability of observing an electron in } x, x + \delta x \tag{4.13}$$

so that we can interpret $\Psi(x,t)$ as a *probability amplitude*. This is the famous probability interpretation of the wave function first proposed by Born on the basis of his own observations of the outcomes of scattering experiments, as well as awareness of Einstein's own inclinations along these lines. Somewhat later, after proposing his uncertainty relation, Heisenberg made a similar proposal.

There are two other important features of this result that are worth taking note of:

- If the detection event can arise in two different ways (i.e. electron detected after having passed through either slit 1 or 2) and the two possibilities remain unobserved, then the total probability of detection is

$$P = |\Psi_1 + \Psi_2|^2 \tag{4.14}$$

  i.e. we add the amplitudes and then square the result.

- If the experiment contains a part that even *in principle* can yield information on which of the alternate paths were followed, then

$$P = P_1 + P_2 \tag{4.15}$$

  i.e. we add the probabilities associated with each path.

What this last point is saying, for example in the context of the two slit experiment, is that, as part of the experimental set-up, there is equipment that is monitoring through which slit the particle goes. Even if this equipment is automated, and simply records the result, say in some computer memory, and we do not even bother to look the results, the fact that they are still available means that we should add the probabilities. This last point can be understood if we view the process of observation of which path as introducing randomness in such a manner that the interference effects embodied in the $\cos \delta$ are smeared out. In other words, the $\cos \delta$ factor – which can range between plus and minus one – will average out to zero, leaving behind the sum of probability terms. It is this that provides an explanation of why it is that bullets do not exhibit interference when passed through a two slit interference pattern. This question goes to the heart of why it is that quantum effects are never seen at the macroscopic level. One answer lies in the fact that the interference fringes for a bullet can be easily shown to be so closely spaced that there is simply impossible to measure the effects: a 10 gm bullet moving at 300 ms^{-1} has a de Broglie wavelength of 2.2×10^{-34}m, and the interference fringe spacing will be correspondingly tiny. But there is more to it than this. Effectively, the position of the bullet is *always* being observed due to its unavoidable interaction with its surrounding environment, which carries the memory of where the particle is positioned at any time, so that it always behaves as a classical particle. Quantum interference effects are being continually 'washed out' by this interaction, so that there is no possibility of observing the fringes as there are none to observe.

4.5   The Fundamental Nature of Quantum Probability

The fact that the results of the experiment performed with electrons yields outcomes which appear to vary in a random way from experiment to experiment at first appears to be identical to the sort of randomness that occurs in the experiment performed with the machine gun. In the latter case, the random behaviour can be explained by the fact that the machine gun is not a very well constructed device: it sprays bullets all over the place. This seems to suggest that simply by refining the equipment,the randomness can be reduced, in principle removing it all together if we are clever enough. At least, that is what classical physics would lead us to believe. Classical physics permits unlimited accuracy in the fixing the values of physical or dynamical quantities, and our failure to live up to this is simply a fault of inadequacies in our experimental technique.

However, the kind of randomness found in the case of the experiment performed with electrons is of a different kind. It is intrinsic to the physical system itself. We are unable

to refine the experiment in such a way that we can know precisely what is going on. Any attempt to do so gives rise to unpredictable changes, via the uncertainty principle. Put another way, it is found that experiments on atomic scale systems (and possibly at macroscopic scales as well) performed under identical conditions, where everything is as precisely determined as possible, will always, in general, yield results that vary in a random way from one run of the experiment to the next. This randomness is irreducible, an intrinsic part of the physical nature of the universe.

Attempts to remove this randomness by proposing the existence of so-called 'classical hidden variables' have been made in the past. These variables are supposed to be classical in nature – we are simply unable to determine their values, or control them in any way, and hence give rise to the apparent random behaviour of physical systems. Experiments have been performed that test this idea, in which a certain inequality known as the Bell inequality, was tested. If these classical hidden variables did in fact exist, then the inequality would be satisfied. A number of experiments have yielded results that are clearly inconsistent with the inequality, so we are faced with having to accept that the physics of the natural world is intrinsically random at a fundamental level, and in a way that is *not* explainable classically, and that physical theories can do no more than predict the probabilities of the outcome of any measurement.

4.6   Wave Mechanics

The version of quantum mechanics based on studying the properties of the wave function is known as wave mechanics, and is the version that first found favour amongst early researchers in the quantum theory, in part because it involved setting up and solving a partial differential equation for the wave function, the famous Schrödinger equation, which was of a well-known and much studied form. As well as working in familiar mathematical territory, physicists and chemists were also working with something concrete in what was in other ways a very abstract theory – the wave function was apparently a wave in space which could be visualized, at least to a certain extent. But it must be borne in mind that the wave function is not an objectively real entity – it turns out that it is no more than one aspect of a more general theory, quantum theory, that is convenient for certain classes of problems, and entirely inappropriate (or indeed inapplicable) to others[4]. Nevertheless, wave mechanics offers a fairly direct route to some of the more important features of quantum mechanics, but more than that it is the formulation of quantum mechanics that is most commonly used in atomic physics, molecular physics, solid state, or in any area of physics in which the position of particles plays a central role.

---

[4]The wave function appears to be a wave in real space for a single particle, but that is only because it depends on x and t, in the same way as, say, the amplitude of a wave on a string can be written as a function of x and t. But, for a system of more than one particle, the wave function becomes a function of two space variables x_1 and x_2 say, as well as t: $\Psi(x_1, x_2, t)$. It then makes no sense to talk about the value of the wave function at some position in space. In fact, the wave function is a wave that 'exists' in an abstract space known as phase space.